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Low-temperature technological combustion of porous systems with forced filtration of a gas reagent

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Abstract—By employing numerical simulation in a one-dimensional adiabatic approximation, the regularities of the steady-state filtration combustion in the modes of accompanying inverse and opposing reaction waves have been studied. With regard to a number of real properties of the system, the relationships have been established between the combustion wave characteristics (temperature, speed of wave propagation and extent of condensed phase conversion in the wave) and the process governing parameters: rate of gas mixture filtration, its temperature and concentration in it of an active component (oxidant). The effect of heat losses on the propagation of these waves is considered in detail. Optimization of the filtration combustion regimes (including those with multiple reaction-wave passage) with respect to the performance of a combustion-based technological process is carried out.

INTRODUCTION

PRACTICAL applications of the waves of exothermal interaction between a porous medium and an active component of a gas mixture being infiltrated into the reaction zone, have a long history going back to the year 1867—the date of the issue of the first patent on ore agglomeration [1]. A substantial increase in practical and scientific interest shown over the last decade in the study of these processes, termed as filtration combustion of porous systems (FCPS), has led to their identification as a separate field of technological combustion.

The range of technological schemes based on the use of filtration combustion (FC) is constantly expanding. In connection with the aforementioned application of filtration combustion in agglomeration, mention must be made of the detailed experimental investigation [2] and of the first theoretical analysis of this phenomenon from the viewpoint of thermal filtration waves in a porous medium [3]. Moreover, this method has traditionally been employed in petroleum extraction through the use of in-pool combustion [4] and for the recovery of catalysts by burning out coke build-ups [5].

Among very important steps in the theory and practice of FC were the works, undertaken by Merzhanov and co-workers, on the development of the technology of self-propagating high-temperature synthesis (SHS) of hard-melting and other non-organic materials [6–18].

The basic scheme of the FC process is as follows [18]. The original porous medium is a condensed fuel, possibly with an inert admixture. An active gaseous component (oxidant) is contained in the gas mixture which then filters through the porous substance or through condensed reaction products and a porous inert skeleton. Gas may be forced through the substance with the aid of some other device (forced filtration), or may filter due to a natural pressure drop between the surrounding gas medium and the reaction zone in which the oxidant is absorbed (natural, self-regulated filtration). After a local 'ignition' of reaction by an external energy source, a self-sustaining reaction wave propagates through the original porous substance at a velocity which is determined by the chemical kinetics and thermal effect of the reaction, and by the heat and mass transfer processes in the porous medium.

Depending on whether the directions of the reaction front and gas filtration coincide or not, two types of FC waves are distinquished: accompanying and opposing.

FCPS Theory has been developed in the fundamental works of Merzhanov, Aldushin *et al.* [8–10, 12–14, 17, 18]. Using essentially approximate analytical methods of classical combustion theory, these authors have resolved the principal problems of the existence and stability of stationary wave solutions, carried out the classification of different types and structures of the FC waves, and investigated the main trends of their propagation.

Combustion with forced filtration of an oxidant lends itself well for use in different technological processes as it provides wide possibilities for controlling the parameters of combustion waves (extent of conversion, temperature at the wave front and rate of its propagation) by varying the input characteristics of a gas flow (velocity, oxidant concentration, temperature).

The specific features of the FC waves with forced gas filtration are largely dictated by that role which an inert gaseous heat carrier plays in these processes. Thus, it allows, if required, a substantial lowering of the combustion temperature as compared with the adiabatic temperature of reaction between an oxidant

NOMENCLATURE

c_j	molar concentration of jth system component
c_{p_j}	molar heat capacity of jth system component
E	reaction activation energy
$f(\eta)$	function of reaction retardation by product
h_j	molar enthalpy of jth system component
L_r	reaction zone width†
L_h	heating zone width†
n	gas-reagent-based reaction order
P	gas mixture pressure
P_1	partial pressure of oxidant (gas reagent)
Q	thermal reaction effect (per mole of a product)
R	universal gas constant
s	determinant of FC regime
T	temperature
u	linear speed of reaction wave propagation
v	linear rate of gas filtration
x	spatial coordinate in the direction of filtration.

Greek symbols

α^*	heat transfer coefficient
η	extent of conversion of treated material
η_f	final conversion extent ($\eta_f = 0-1$)
λ	effective thermal conductivity of system
μ	stoicheiometric gas-reagent-based reaction factor
Φ	reaction rate.

Subscripts

1, 2, 3, 4	refer, respectively, to an active component of filtered gas (reactant), its inert component, porous material and reaction product
0	refers to initial values of parameters (for a gas phase, designates the 'inlet' parameters in treated material layer)
c	relates to values of parameters in reaction zone.

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† The width of reaction zone is defined as the region in which the rate of reaction is equal to, or much greater than, 0.1 of its maximum value, and the heating zone width is defined as a region in which the temperature falls e times as compared with the temperature at the level of the left boundary of reaction zone.

and condensed substance. This feature of FC waves becomes their main advantage in those situations when there are several concurrent, thermally-activated, highly exothermal reactions in a system, while the optimum (as to the performance) temperature of the basic reaction is much below its thermodynamic equilibrium temperature; this explains the term 'low-temperature combustion' in the title of the paper.

The FC theory [8–10, 18] has proved the feasibility of structurally different thermal reaction waves, the following two of which are of interest for the technological process considered here: the accompanying inverse wave (with preheated initial substance and cooled product) and the opposing wave (Fig. 1). Their specific application requires more thorough theoretical investigation since the analytical method, used in the aforementioned works, does not provide a simultaneous account for the entire set of inherent properties. These are: the non-zero reaction order, temperature-dependent thermophysical properties of substances partaking in reaction (reactants and products), the sensitivity of the effective thermal

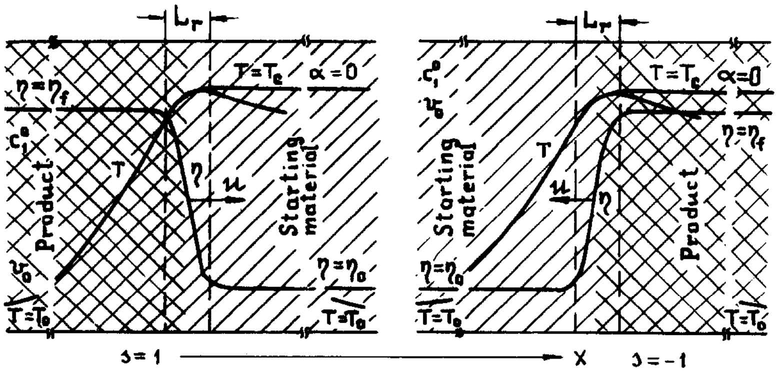

FIG. 1. Calculation scheme.

conductivity of the system to the extent of condensed reagent conversion, reaction retardation by the product, and the effect of heat losses on the process.

At the same time, accounting for these properties, as follows from work [19] and is illustrated below, transforms the available analytical relations not only quantitatively, but in some cases also qualitatively.

These facts have necessitated investigations in the FC field by employing numerical simulation methods, the basic and primarily new results of which are considered in this paper.

STATEMENT AND SOLUTION OF THE PROBLEM

In stating the problems of the FC theory [18] use is made of the continuum approach, with the medium being homogeneous as to temperature and concentration. The chemical heterogeneity of the system is taken into account by the kinetic equation in which the rate of reaction depends on both the oxidant concentration and the extent of condensed substance conversion, and vanishes as soon as any of the reactants burns out. The system of heat and mass transfer equations in the one-dimensional statement, which describes the process in the system of coordinates moving with a reaction wave (Fig. 1), is similar to that used by Aldushin and Seplyarsky [8–10] (the only difference being that one of the variables has been modified to allow the formulation without reference to the direction of wave propagation [20]):

$$\frac{\mathrm{d}}{\mathrm{d}x}[(c_1h_1+c_2h_2)v-s(c_3h_3+c_4h_4)u] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\lambda(\eta)\frac{\mathrm{d}T}{\mathrm{d}x}\right]+Qc_3^0\Phi-\alpha^*(T-T_0) \quad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[(c_1+c_2)v] = -\mu c_3^0\Phi \quad (2)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(c_1v) = -\mu c_3^0\Phi \quad (3)$$

$$-u\frac{\mathrm{d}\tilde{\eta}}{\mathrm{d}x} = \Phi(T, \eta, P_1), \quad (4)$$

where

$$\tilde{\eta} = \frac{1-s}{2}\eta_{\mathrm{f}}+s\eta-\frac{1+s}{2}\eta_0 \quad (5)$$

$$\eta = \frac{c_3^0-c_3}{c_3^0} = \frac{c_4}{c_3^0} \quad (6)$$

$$\Phi = k\exp[-E/RT]f(\eta)P_1^n = k\exp[-E/RT]f(\eta)(RT)^nc_1^n \quad (7)$$

$$h_4 = h_3+\mu h_1, \quad h_j = \int_{T_0}^{T} c_{\mathrm{p}_j}\,\mathrm{d}T \quad (8)$$

$$c_1+c_2 = C = P/RT, \quad (9)$$

$s = 1$ in the case of an accompanying wave and $s = -1$ in the case of an opposing wave.

The boundary conditions for the system of equations (1)–(4) have the form:

$$x \to -\infty: T = T_0; c_1 = c_1^0, \quad \tilde{\eta} = \tilde{\eta}_{\mathrm{f}} = \eta_{\mathrm{f}}-\eta_0, \quad \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\mathrm{d}\tilde{\eta}}{\mathrm{d}x} = 0 \quad (10)$$

$$x \to +\infty: T = T_0(T_{\mathrm{c}} \quad \text{at} \quad \alpha^* = 0), \quad c_1 = 0, \quad \tilde{\eta} = 0, \quad \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\mathrm{d}\tilde{\eta}}{\mathrm{d}x} = 0. \quad (11)$$

In the present statement of the problem it is assumed that the diffusion Péclet number $Pe_{\mathrm{D}} = vL_{\mathrm{r}}/D \gg 1$, and this, in turn, allows one to neglect the diffusion component of the oxidant flux. Furthermore it is assumed that: the speed of gas filtration is much higher than the speed of reaction wave propagation ($v \gg u$); the size of the porous medium in the direction of filtration is much greater than the size of zones of reaction and heating; the pressure drop over the specimen is small compared with the absolute pressure at the entrance into it ($\Delta P/P \ll 1$); the outbreak of the oxidant through the reaction zone is absent; the temperature difference in the specimen section normal to the direction of filtration is small and the reaction front is little distorted; the chemical reaction of product formation can be characterized by one stoicheiometric coefficient (μ); the thermal effect of the reaction is independent of temperature, i.e. the thermal capacity of the product is additive with respect to the thermal capacities of the original condensed substance and oxidant.

The integral relations following from the above system of equations can now be determined.

The integration of the transport equations of gas components (2) and (3), accounting for equation (4) from $-\infty$ to x, yields relations of the form

$$c_2v = c_2^0v_0 \quad (12)$$

$$c_1v-c_1^0v_0 = \mu c_3^0u(\tilde{\eta}-\tilde{\eta}_{\mathrm{f}}). \quad (13)$$

From equation (13) with boundary condition (11) follows one of the main integral FC relations characterizing the oxidant balance:

$$u = \frac{c_1^0v_0}{\mu c_3^0\tilde{\eta}_{\mathrm{f}}}. \quad (14)$$

The state equation (9) and integrals (12)–(14) make it possible to obtain the relationship between the local values of oxidant concentration, temperature and extent of conversion:

$$c_1 = C^0\frac{T_0}{T}\left[1+\frac{\tilde{\eta}_{\mathrm{f}}}{\tilde{\eta}}\left(\frac{C^0}{c_1^0}-1\right)\right]^{-1}. \quad (15)$$

Changing relation (15) to a relative oxidant concentration $\xi = c_1/C$ gives

$$\xi = \left[1+\frac{\tilde{\eta}_{\mathrm{f}}}{\tilde{\eta}}(1/\xi_0-1)\right]^{-1}. \quad (16)$$

In an adiabatic approximation ($\alpha^* = 0$), it is possible to use two more integral relations characteristic for the considered FC problems. Using relations (5), (6), (8) and integrals (12)–(14) for the transformation of the LHS of energy equation (1) and integrating the latter once from $-\infty$ to x, yields

$$c_2^0 v_0 h_2 - suc_3^0 h_3 + \mu c_3^0 u h_1 \left(\frac{1-s}{2}\tilde{\eta}_f - s\eta_0\right) = \lambda(\eta)\frac{dT}{dx} - Qc_3^0 u(\tilde{\eta} - \tilde{\eta}_f). \quad (17)$$

After the substitution of the boundary condition at $+\infty$ into relation (17), it is possible to find

$$\eta_f = \eta_0 + \left[\frac{Ph_2^c}{RT_0 c_3^0}\cdot\frac{v_0}{u} - s(h_3^c + \mu h_1^c \eta_0)\right] \Bigg/ \left[Q + \mu\left(h_2^c - \frac{1-s}{2}h_1^c\right)\right]. \quad (18)$$

Relations (14) and (18) yield a unique correlation between the relative concentration of an oxidant at the inlet to the system and final extent of conversion of the material treated (at the given temperature in the reaction zone T_c):

$$\xi_0 = \mu h_2^c \Bigg/ \left[\mu\left(h_2^c - \frac{1-s}{2}h_1^c\right) + Q + s(h_3^c + \mu h_1^c \eta_0)/(\eta_f - \eta_0)\right]. \quad (19)$$

where $h_j^c = a_j(T_c - T_0) + b_j(T_c^2 - T_0^2) - c_j(1/T_c - 1/T_0)$, and a_j, b_j, c_j are the constants in the formula for computing the temperature dependence of the heat capacity of the jth system species.

This relation is of extreme importance for practical application: it allows one to evaluate rather simply the oxidant concentration which ensures the required technological parameters of the process (T_c and η_f).

It is convenient to carry out the numerical solution of the problem in dimensionless variables determined as

$$\Theta = \frac{E}{RT_*^2}(T - T_*), \quad \tilde{\xi} = \xi/\xi_*, \quad \tilde{x} = x/x_*, \quad \tilde{c}_{p_j} = c_{p_j}/c_{p_j}^0, \quad \tilde{\lambda} = \lambda/\lambda_*. \quad (20)$$

Taking into account the integral relations obtained earlier, the dimensionless system of equations and boundary conditions takes the form:

$$\gamma\left\{\tilde{v}_0\left[\tilde{c}_{p_2} - \xi_*\tilde{\xi}_0\left(\tilde{c}_{p_2} - \left(\frac{1-s}{2} - s\frac{\eta_0}{\tilde{\eta}_f}\right)\Psi\tilde{c}_{p_1}\right)\right] - s\tilde{c}_{p_3}\tilde{u}\right\} \times \frac{d\Theta}{d\tilde{x}} = \frac{d}{d\tilde{x}}\left(\tilde{\lambda}\frac{d\Theta}{d\tilde{x}}\right) + \varphi(\Theta, \eta, \tilde{\xi}) - \kappa(\Theta - \Theta_0), \quad (21)$$

$$-\tilde{u}\frac{d\tilde{\eta}}{d\tilde{x}} = \varphi(\Theta, \eta, \tilde{\xi}) \quad (22)$$

$$\varphi = f(\eta)\tilde{\xi}^n \exp\left[\frac{\Theta}{1 + \beta\Theta}\right] \quad (23)$$

$$\xi_*\tilde{\xi}_0\tilde{v}_0 = \delta\tilde{\eta}_f\tilde{u} \quad (24)$$

$$\tilde{\xi} = \tilde{\eta}/\xi_*[\tilde{\eta} + \tilde{\eta}_f(1/\xi_0 - 1)]. \quad (25)$$

In the absence of heat losses ($\kappa = 0$)

$$\tilde{\xi}_0 = \tilde{h}_2^c/\xi_*\left[\tilde{h}_2^c - \frac{1-s}{2}\Psi\tilde{h}_1^c + \frac{1}{\delta\gamma} + s\left(\frac{1}{\delta}\tilde{h}_3^c + \Psi\tilde{h}_1^c\eta_0\right)\Bigg/\tilde{\eta}_f\right], \quad \tilde{h}_j^c = \int_{\Theta_0}^{\Theta_c}\tilde{c}_{p_j}\,d\Theta \quad (26)$$

$$\tilde{x} \to -\infty : \Theta = \Theta_0, \quad \tilde{\xi} = \tilde{\xi}_0, \quad \tilde{\eta} = \tilde{\eta}_f = \eta_f - \eta_0, \quad \frac{d\Theta}{d\tilde{x}} = \frac{d\tilde{\eta}}{d\tilde{x}} = 0 \quad (27)$$

$$\tilde{x} \to +\infty : \Theta = \Theta_0(\Theta_c \quad \text{at} \quad \kappa = 0), \quad \tilde{\xi} = 0, \quad \tilde{\eta} = 0, \quad \frac{d\Theta}{d\tilde{x}} = \frac{d\tilde{\eta}}{d\tilde{x}} = 0. \quad (28)$$

Equations (21)–(28) contain the following dimensionless parameters:

$$\tilde{v}_0 = v_0/v_*, \quad \tilde{u} = u/u_*, \quad \tilde{\xi}_0 = \xi_0/\xi_*, \quad \kappa = \frac{\alpha^*\gamma t_*}{c_3^0 c_{p_3}^0}, \quad \gamma = \frac{RT_*^2/E}{Q/c_{p_3}^0} \quad (29)$$

$$\beta = RT_*/E, \quad \Psi = c_{p_1}^0/c_{p_2}^0, \quad \delta = \mu\cdot c_{p_2}^0/c_{p_3}^0.$$

The non-dimensional scales are determined by the expressions

$$x_* = \sqrt{a\gamma t_*}, \quad (30)$$

$$u_* = \sqrt{a\gamma/t_*}, \quad (31)$$

where $a = \lambda_0/c_3^0 c_{p_3}^0$ is the effective thermal diffusivity of a porous material in the initial state,

$$v_* = u_*(c_3^0 c_{p_3}^0 RT_0/Pc_{p_2}^0), \quad (32)$$

$$\xi_* = \delta\left[s - \frac{1}{\gamma\Theta_0} + \delta\left(1 - \frac{1-s}{2}\Psi\right)\right]^{-1} \quad (33)$$

$$\lambda_* = \lambda_0 \quad (34)$$

$$t_* = \frac{1}{k(P\xi_*)^n \exp(-E/RT_*)}. \quad (35)$$

The scale of concentration ξ_* is chosen to be equal to an initial oxidant concentration ξ_0 which provides the final extent of conversion $\eta_f = 1$ at the temperature in the reaction zone $T_c = T_*$, with constant heat capacities of the system components ($\tilde{c}_{p_j} = 1$) and zero initial extent of conversion ($\eta_0 = 0$), and in the absence of heat losses.

It should be noted that the choice of T_* and T_0 uniquely determines the value of ξ_* (at the given $E, Q, c_{p_j}^0$) which is independent of such kinetic parameters as k and n, and of the form of the function $f(\eta)$. As to the choice of the characteristic temperature T_*, it is convenient that it be assumed equal to the optimal temperature for the given technological process (for example, optimal with respect to the production of the target product and freezing of concurrent reactions).

The FC wave parameters (η_f, $\tilde{u}$, Θ_c), as functions of the inlet parameters of the system ($\tilde{v}_0$, $\tilde{\xi}_0$, Θ_0, κ) can be found by numerical solution of the above system of equations with the aid of the algorithm given in ref. [20].

In calculations (unless otherwise specified) a model system was used which is characterized by the following dimensionless parameters: $\gamma = 1.47 \times 10^{-3}$; $\beta = 0.041$; $\Theta_0 = -15.5$; $\delta = 1.22$; $\Psi = 1.5$; $n = 1/2$; $f(\eta) = (1-\eta)^l$; $\tilde{\lambda} = 1 - m\eta$.

RESULTS OF NUMERICAL SIMULATION OF FC AND THEIR DISCUSSION

Before going over to the discussion of the laws governing the FC, the importance of accounting for the real characteristics of the system will be illustrated and, in particular, of the dependence of its effective thermal conductivity on the extent of conversion of the original porous material and reaction retardation by the product. In Fig. 2 the dependence of the extent of conversion on the gas filtration rate at a constant temperature in the combustion wave Θ_c is given for different model functions $\lambda(\eta)$ and $f(\eta)$. It is seen how quantitatively, and in some cases qualitatively, the 'approximate' relations (curves 2–7) differ from the 'exact' solution (1).

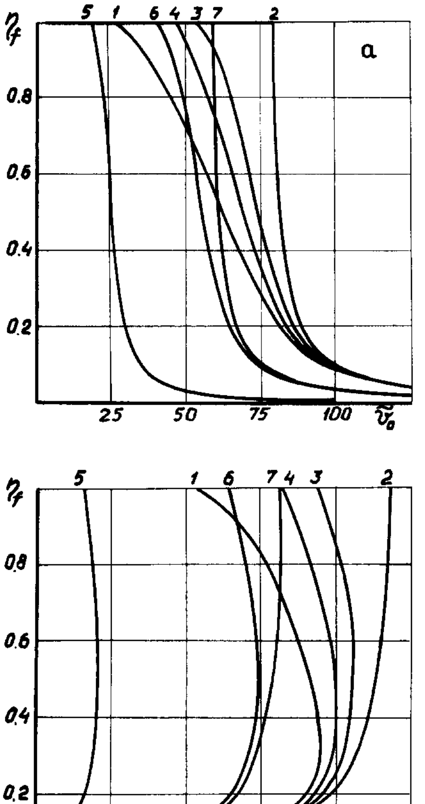

FIG. 2. Dependence of the final extent of treated material conversion on gas mixture filtration rate in the regimes of accompanying inverse (a) and opposing (b) FC waves. ($\Theta_0 = -15.5$, $\Theta_c = 0.74$). 1: $m = 0.9$, $l = 1/2$; 2: $m = 0$, $l = 0$; 3: $m = 0$, $l = 1/2$; 4: $m = 0.9$, $l = 0$; 5: $m = 0$, $l = 1/2$; 6: $m = 0$, $l = 1/2$; 7: $m = 0$, $l = 0$. Curves 1, 2, 3, 4, λ_0(W m^{-1} K^{-1}) = 0.5; 5, 0.06; 6, 7, 0.28.

1. *Trends in technological combustion in the mode of an accompanying inverse steady-state FC wave (adiabatic approximation)*

When using the FC for low-temperature synthesis, of primary interest are the characteristics of the reaction wave that correspond to such combinations of the governing parameters which ensure the prescribed temperature in the hot zone. As an example, Fig. 3 presents a family of such 'isothermal' characteristics of the FC process.

A decrease in the filtration rate, provided the combustion temperature Θ_c remains constant, requires a corresponding increase in the oxidant concentration and leads to an increase in the extent of conversion with simultaneous decrease in the speed of wave propagation. At some value of $\tilde{v}_0 = \tilde{v}_0^*$ (respectively, $\tilde{\xi}_0 = \tilde{\xi}_0^*$), the original condensed substance converts completely to the product after the passage of the reaction front (the occurrence of the so-called complete conversion regime CCR). A further decrease in the rate of filtration into the region $\tilde{v}_0 < \tilde{v}_0^+$ at $\tilde{\xi}_0 = \tilde{\xi}_0^+$ does not change the prescribed temperature in the reaction zone Θ_c, but the speed of combustion wave propagation $\tilde{u}$ in this case decreases linearly with $\tilde{v}_0$ according to the integral relation (14) at $\eta_f = 1$.

At high gas flow rates the extent of conversion is not high, while the speed of reaction wave propagation acquires large values (by two orders of magnitude exceeding the speed of wave of the CCR, Θ_c = const.). The oxidant concentrations in gas flow that ensure, at high filtration rates, the prescribed combustion temperature Θ_c, are much smaller than those of the CCR.

Numerical calculations show that reaction retardation by the product, its fractional order ($n < 1$) and a drop in the thermal conductivity of the system with an increase in the conversion extent lead to a smoother dependence of the wave characteristics on the gas filtration rate. Because of the effect of these factors, the value of the filtration rate $\tilde{v}_0^*$, at which the CCR sets in, and the corresponding speed of reaction wave propagation decreases 3–4 times as compared with the values of these quantities in an ideal system [$f(\eta) = 1$, $\lambda = \lambda_0$, $n = 0$].

From the practical point of view, of the greatest interest are the reaction modes near $\tilde{v}_0^*$, especially in some region to the left of it, where two parameters of the wave ($\eta_f = 1$ and Θ_c) are insensitive to a change in the filtration rate (at $\tilde{\xi}_0 = \xi_0^*$ = const.).

The effects of each of the inlet parameters of a gas flow on the characteristics of the technological combustion process are considered at length in ref. [21]. Note only that due to the reaction rate dependence on the extent of conversion and oxidant concentration, the

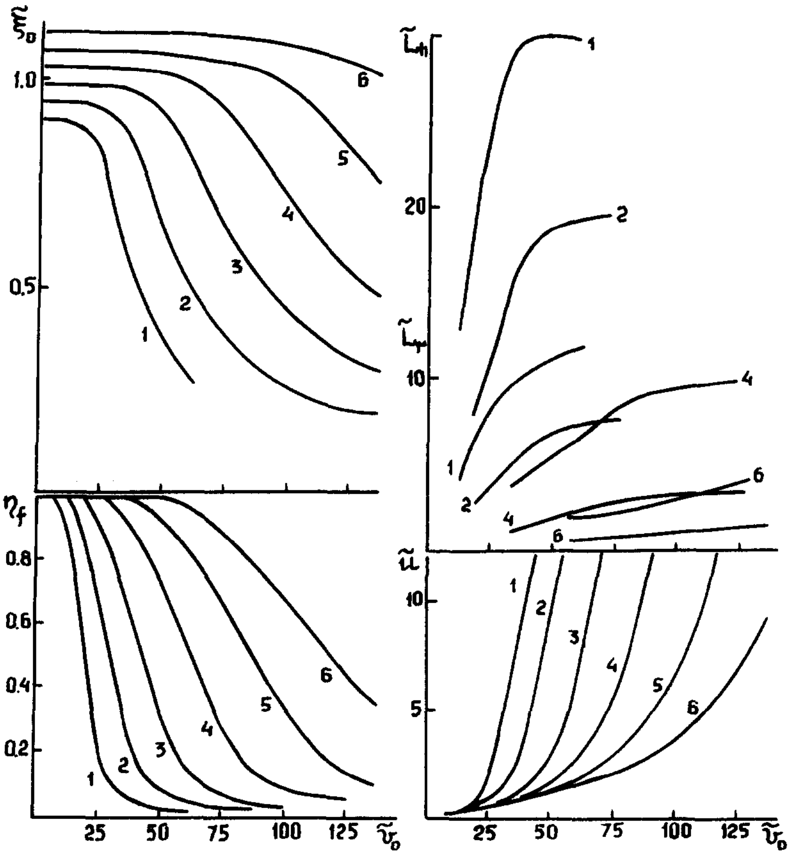

FIG. 3. Dependence of the isothermal characteristics of the FC process in the accompanying inverse wave state ($\eta_f, \xi_0, \tilde{u}, L_r, L_h$) on gas filtration rate ($\tilde{v}_0$). Θ_c: $-1.48(1)$; $-0.74(2)$; $0(3)$; $0.74(4)$; $1.48(5)$; $2.21(6)$; $\Theta_0 = -15.5$.

temperature in the combustion wave is not a single-valued function of the oxidant flow rate as followed from the Zel'dovich–Frank-Kamenetsky integral in the absence of reaction retardation by the product and reaction zero order. It turns out that the value of Θ_c depends not only on the power of heat generation (i.e. quantity of oxidant supplied per unit time), but also on the manner in which this power is supplied—at the expense of oxidant concentration or filtration rate. A high oxidant concentration, and a correspondingly low rate of filtration, lead to a higher temperature in the reaction wave and, as a consequence, to a greater extent of condensed component conversion at a slower speed of wave propagation. Thus, for example, at the values of the inlet parameters $\Theta_0 = -15.5$, $\tilde{v}_0 = 50$, $\tilde{\xi}_0 = 1.06$ ($\tilde{v}_0\tilde{\xi}_0 = 53$), the outlet characteristics of the process are: $\Theta_c = 1.27$, $\eta_f = 0.9$, $\tilde{u} = 1.25$; at the same time when $\Theta_0 = -15.5$, $\tilde{v}_0 = 71$, $\tilde{\xi}_0 = 0.75$ ($\tilde{v}_0\tilde{\xi}_0 = 53$), one obtains $\Theta_c = 0.12$, $\eta_f = 0.11$, $\tilde{u} = 10.3$. The dimensions of the reaction zones also differ substantially for these two regimes: $\tilde{L}_r \approx 0.91$ for the first and $\tilde{L}_r \approx 4.37$ for the second.

The analysis carried out in ref. [21] has shown an extremely strong dependence of temperature of the reaction wave and of the conversion extent on the oxidant concentration and, to a lesser extent, on the gas filtration rate. This imposes special requirements on the accurate sustainment of the inlet parameters during the process.

2. *Trends in the technological combustion in the mode of an opposing steady-state FC wave (adiabatic approximation)*

First of all, some corollaries following from integral relation (19) will be noted. First, at the given temperatures Θ_0 and Θ_c the region of the existence of opposing waves lies in the range of higher oxidant concentrations than for an accompanying wave. The physical reason for this is an additional withdrawal of heat in an opposing wave by an originally cold substance coming to the reaction zone. This also explains the existence of the minimum extent of condensed substance conversion in the mode of an opposing wave at the fixed values of Θ_0 and Θ_c. It follows from relation (19) that

$$\eta_{f,\min} = \frac{Q\eta_0 + h_3^c}{Q - \mu h_1^c}. \tag{36}$$

The $\eta_{f,\min}$ is achieved at the 100% concentration of an oxidant in a gas flow.

The dependence of the characteristics of an opposing wave and of oxidant concentration $\tilde{\xi}_0$ on the gas

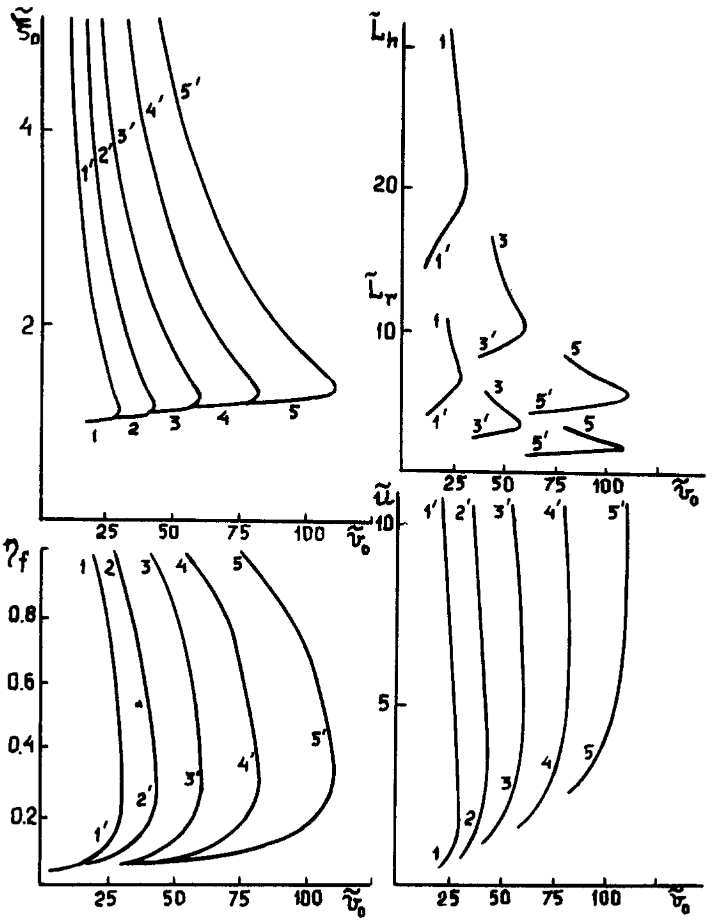

FIG. 4. Dependence of the isothermal characteristics of FC process in the opposing wave regime (η_f, $\tilde\xi_0$, $\tilde u$, $\tilde L_r$, $\tilde L_h$) on gas filtration rate ($\tilde v_0$). Θ_c: -0.74 (1–1′); 0(2–2′); 0.74(3–3′); 1.48(4–4′); 2.21(5–5′); $\Theta_0 = -15.5$.

filtration rate, corresponding to the prescribed values of Θ_0 and Θ_c, are presented in Fig. 4. The fact that attracts one's attention is the non-uniqueness of these relations: two opposing waves with the same combustion temperatures Θ_c but with different conversion extents, speeds of propagation and dimensions of characteristic zones, can be realized at the same values of Θ_0 and $\tilde v_0$, but at different oxidant concentrations $\tilde\xi_0$. To the rising branch of $\eta_f(\tilde v_0)$ there corresponds also the rising branches of $\tilde L_r(\tilde v_0)$, $\tilde L_h(\tilde v_0)$ and falling branches of $\tilde u(\tilde v_0)$ and $\tilde\xi_0(\tilde v_0)$. As an example, quantitative characteristics of two such waves are listed in Table 1.

The non-linear character of the phenomenon complicates a detailed physical interpretation of the data obtained, but the analysis of Table 1 makes it possible to establish a certain correlation between the parameters of the two waves. At higher oxidant concentrations the reaction is localized in a narrow region and its rate is high (meaning a high specific rate of heat generation related to the reaction zone width). To preserve the given combustion temperature level, the wave should have rather a high speed (then the heat removal due to the original material heating increases). This results in a mode with a high speed of propagation and low conversion extent. Simultaneously, the mean value of thermal conductivity of the condensed phase increases and reaction retardation by the product

Table 1. Characteristics of the opposing wave as functions of the inlet system parameters

Inlet parameters			Characteristics of an opposing wave					
Θ_0	$\tilde v_0$	$\tilde\xi_0$	Θ_c	η_f	$\tilde u$	$\tilde L_r$	$\tilde L_h$	Φ/Φ_0
−15.5	37.1	1.05	0.15	0.83	1.11	5.87	18.03	0.24
−15.5	37.1	1.66	0.15	0.11	13.15	3.60	11.43	0.46

decreases. In the opposite case (at lower concentrations), a low-rate mode is realized which has a large extent of conversion, low thermal conductivity of the product and strong retardation of the reaction rate.

The effects of each of the inlet parameters of the gas flow on the process characteristics are investigated in detail in ref. [22]. Of special interest for the FC regime considered is the sensitivity of the outlet characteristics of the process to the oxidant concentration at constant values of two other parameters of the filtered gaseous mixture: Θ_0 and $\tilde{v}_0$ (Fig. 5). To high values of $\tilde{\xi}_0$ there correspond the dependences typical of an 'ideal' opposing wave [9] $(n = 0, l = 0, m = 0)$: with a decrease in the oxidant concentration, the conversion extent increases while the temperature and propagation speed decrease. However, with a further decrease in $\tilde{\xi}_0$, the character of the dependences changes: the region of non-uniqueness appears where for one set of inlet parameters of the gaseous mixture there are two steady-state solutions that determine the two waves with different characteristics.

On the right boundary of the non-uniqueness region the conversion extent in one of the solutions is equal to unity and in the other is less than unity. When $\tilde{\xi}_0$ approaches the left boundary of the non-uniqueness region, the parameters of the both solutions get closer and, finally, merge at the boundary. For smaller oxidant concentrations the steady-state one-dimensional solutions are absent.

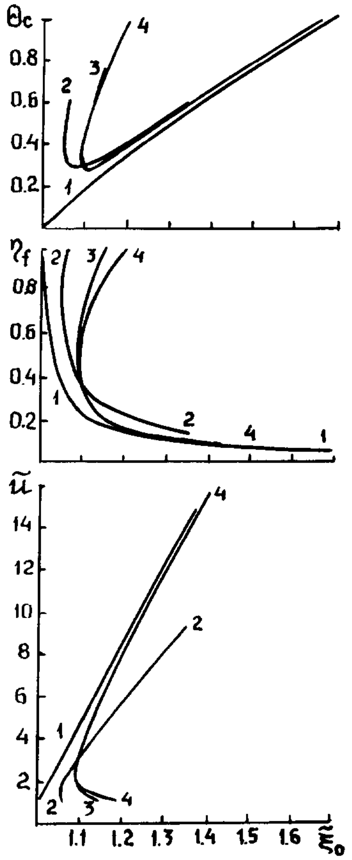

FIG. 5. Dependence of the opposing FC wave characteristics $(\eta_f, \tilde{u}, \Theta_c)$ on relative gas–reagent concentration $(\tilde{\xi}_0)$. $\beta = 0.04$; $\gamma = 1/300$; $\tilde{v}_0 = 60$; 1, $\Theta_0 = -5$, $m = 0$; 2, $\Theta_0 = -10$, $m = 0.9$; 3, $\Theta_0 = -5$, $m = 0.9$; 4, $\Theta_0 = -5$, $m = 0.99$.

The appearance of the non-uniqueness region of opposing FC wave propagation is entirely governed by the reaction rate dependence on the oxidant concentration and on the extent of condensed reactant conversion and also by a decrease in the thermal conductivity with an increasing conversion extent.

For highlighting the physical situation, the following reasonings are suggested. Consider the solution branch which corresponds to high oxidant concentrations (low values of η_f and high wave propagation speeds). A decrease of the concentration on this branch first leads to a decreasing temperature in the reaction wave, since the quantity of heat, generated in the reaction zone, decreases. The speed of wave propagation also decreases, while the conversion extent increases. The change in the wave temperature behaviour with a further decrease in concentration can be explained as follows. The heat, generated in the reaction zone, is carried away by the incoming gas flow and is spent to heat a cold condensed reactant coming into the zone at the speed $\tilde{u}$. As the extent of the condensed reagent conversion increases, the thermal conductivity of the system decreases and, as a result, the wave speed decreases (proportional to $\sqrt{\lambda}$). This leads to less heat removal from the reaction zone, with a decrease in heat removal being higher than that due to a smaller oxidant concentration. The temperature in the reaction wave starts to rise and then the combustion terminates (at the level of minimum concentration corresponding to the onset of the opposing FC wave region). The waves with high conversion extents, corresponding to the other solution branch, are characterized by higher temperature and lower speed of propagation.

The reaction retardation by the product formally influences the solution in exactly the same way as a drop in the thermal conductivity with an increasing conversion extent. The increase of these mutual effects expands the region of non-uniqueness.

Numerical simulation of the unsteady-state processes of ignition and combustion of finite systems [23] has discovered the instability of the steady-state solution at high values of η_f, which impedes the practical realization of these FC regimes.

3. *The effect of heat losses on the process of low-temperature technological combustion*

The complex character of processes occurring in the FC makes it necessary to invoke these or other approximations and assumptions among which probably the most important is the assumption of the adiabatic nature of the system (see above). At the same time, the actual realization of the FC always involves certain heat losses. It was shown earlier [8, 18, 24] that in many FC processes (opposing and accompanying waves with natural filtration of an oxidant, FC of dissociating systems, etc.) the effect of heat losses on the macrokinetic characteristics of these processes is very great and turns out to be more complicated than with slow combustion of gaseous mixtures [25]. The heat losses play an especially important role in low-temperature FC regimes with forced gas filtration due to extremely low propagation rates of corresponding reaction waves.

In what follows, results are presented on the effect of heat losses on the FC process investigated in a one-dimensional approximation. For the first time this approach was used in ref. [25]. In the present case this approach means that the temperature drop in the reaction zone in the direction normal to that of gas filtration does not exceed one/two characteristic temperature intervals, i.e. the reaction rate on the side surface of the material is sufficiently high for the conversion to be able to involve the entire specimen and for the reaction front to be slightly curved.

A rigorous transition to a one-dimensional approximation of the two-dimensional problem can be accomplished following ref. [26] by averaging over the transverse coordinate y with the weight factor λ_T which is the first eigenfunction of the equation

$$\frac{d^2\vartheta}{d\tilde{y}}+\lambda_T\vartheta = 0$$

with the boundary conditions

$$\left.\frac{d\vartheta}{d\tilde{y}}\right|_{\tilde{y}=0} = 0, \quad \left.\frac{d\vartheta}{d\tilde{y}}\right|_{\tilde{y}=1} = -Bi\vartheta.$$

Here, $\tilde{y} = y/H$, $Bi = \alpha H/\lambda$, and α is the coefficient of heat transfer on the side surface of the specimen, H is the specimen half-thickness.

The eigenvalue λ_T satisfies the equation $\sqrt{\lambda_T}\tan\sqrt{\lambda_T} = Bi$. After the averaging, the contribution of the transverse heat transfer into the energy equation is determined by the expression

$$\lambda\overline{\frac{d^2T}{dy^2}} = \lambda\frac{\lambda_T}{H^2}(T-T_0).$$

Whence $\alpha^* = \lambda(\lambda_T/H^2)$ and at sufficiently small values of Bi ($Bi < 0.1$) when $\lambda_T \to Bi$

$$\alpha^* \approx \lambda\frac{Bi}{H^2} = \frac{\alpha}{H}. \tag{37}$$

Numerical simulation of the effect of heat losses on the types of FC waves considered here was carried out for a model system characterized by the following values of the dimensionless parameters: $\gamma = 1/300$, $\beta = 0.04$, $\Theta_0 = -10$, $\delta = 1$, $\Psi = 1$. Two versions with different combinations of the rest parameters were investigated. The first version, which is the main one for the further discussion of general trends in the FC wave propagation with heat losses, was governed by the zero-order reaction, the absence of reaction retardation by its product and by the constant thermal conductivity of the system: $n = 0$, $l = 0$, $m = 0$. In the second version $n = 1/2$, $l = 1/2$, $m = 0.9$. For convenience, the results of calculations based on the parameters $\tilde{u}$ and $\tilde{v}_0$ are presented in the form of ratios $\tilde{u}/\hat{u}$ and $\tilde{v}_0/\hat{v}_0$, where $\hat{u} = \sqrt{2}$ and $\hat{v}_0 = \delta\sqrt{2}/\xi_*$ are the parameters of the FC waves at $\xi_0 = 1$, $\eta_f = 1$, $\Theta_c = 0$ obtained from the approximate solution of the problem, which corresponds to the main version in the absence of heat losses [9, 10]:

$$u\eta_f = \sqrt{2}u_* = \sqrt{2a\gamma/t_*}.$$

Analysis of the relationships [27] between the outlet characteristics of FC waves and the gas filtration rate at the fixed values of the remaining inlet parameters, given partially in Figs. 6 and 7, allows the following conclusions to be drawn:

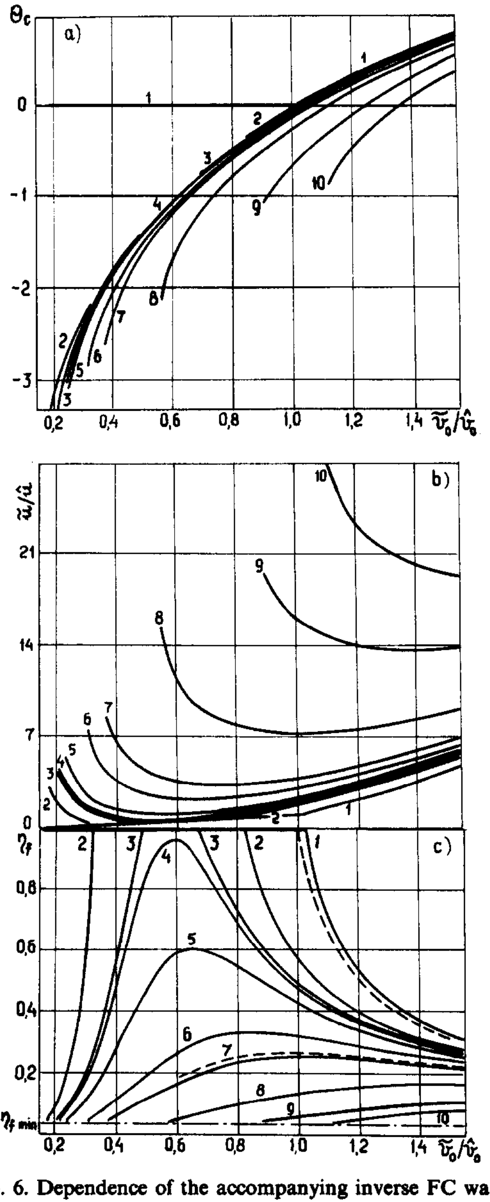

FIG. 6. Dependence of the accompanying inverse FC wave characteristics (Θ_c—a, $\tilde{u}$—b, η_f—c) on gas filtration rate. $\xi_0 = 1$; κ is equal to: 1, 0; 2, 0.0003; 3, 0.0004; 4, 0.00042; 5, 0.0005; 6, 0.00075; 7, 0.001; 8, 0.002; 9, 0.004; 10, 0.006. ---Zel'dovich–Frank-Kamenetsky approximation.

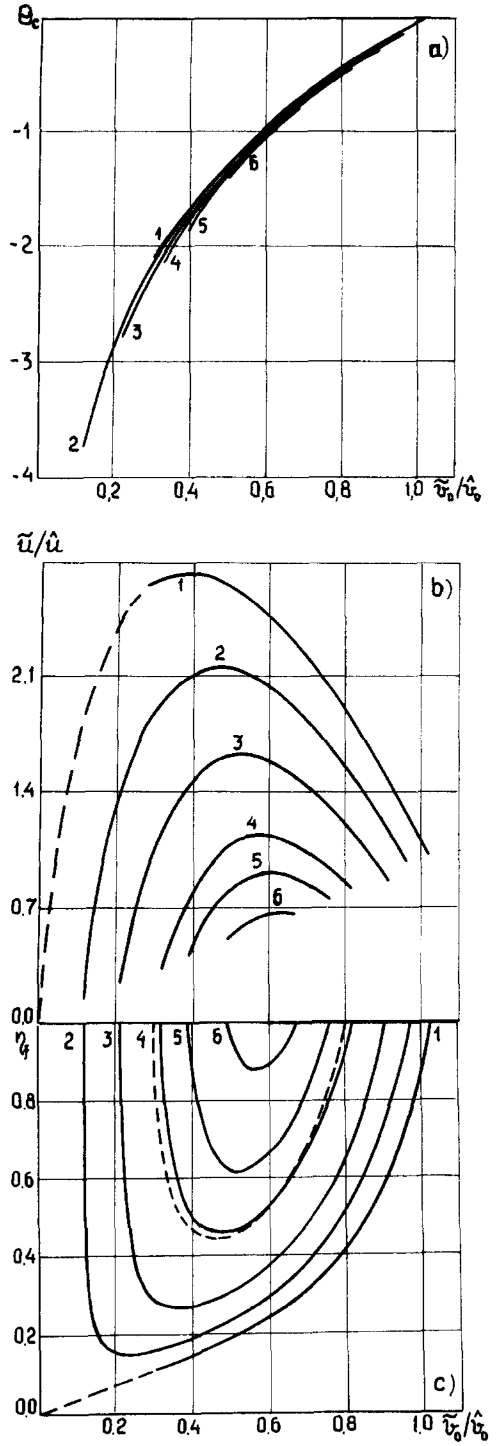

FIG. 7. Dependence of the opposing FC wave characteristics (Θ_c—a, $\tilde{u}$—b, η_f—c) on gas filtration rate. $\xi_0 = 1$; κ is equal to: 1, 0; 2, 0.0001; 3, 0.0002; 4, 0.0003; 5, 0.00035; 6, 0.0004. --- Zel'dovich–Frank-Kamenetsky approximation.

Accompanying inverse FC waves. When $\kappa > 0$, a new branch of the solution with $\eta_f < 1$ appears, which is located in the region of small filtration rates. When $\kappa \to 0$, it degenerates into the ordinate axis on the plot $\eta_f(\tilde{v}_0)$ (Fig. 6). When $\kappa < \kappa_{cr_1}$, the RHS ('high-rate') and LHS ('low-rate') branches of the solution with $\eta_f < 1$ are separated by the region of complete conversion ($\eta_f = 1$). When $\kappa \geqslant \kappa_{cr_1}$, the left and the right branches of the solution with $\eta_f < 1$ join; the function $\eta_f(\tilde{v}_0)$ has a maximum less than 1.

The thermal balance in the accompanying inverse FC wave with an increasing coefficient of heat losses κ and at the prescribed initial parameters $\tilde{v}_0$, ξ_0 and Θ_0 is brought about at the expense of a decrease in heat removal from the reaction zone (a consequence of combustion temperature decrease, Fig. 6a), and at the expense of the heat supplied to the reaction zone by a heated porous material as a result of increase in the wave propagation rate (Fig. 6b); the latter is accompanied by a decrease in the extent of material conversion (Fig. 6c). Its minimum value $\eta_{f,\min} = \xi_0/(1-\xi_0)\delta$ corresponds to the equality $\tilde{u} = \tilde{v}_0(1-\xi_0)$. In the absence of heat losses, the approach of the propagation speeds of the reaction wave and of thermal perturbation $\tilde{u} \to \tilde{u}_h = \tilde{v}_0(1-\xi_0)$ means that the combustion temperature tends to infinity, i.e. there is a kind of a 'resonance'. A further increase in the combustion wave speed must lead to a transition to the normal front structure and to the development of the finite combustion temperature. Thus, on the attainment of a certain value of κ_{cr_2} (specific for each velocity $\tilde{v}_0$) the heat losses, even in the 'resonance' regime, will exceed the heat generation determined by the oxidant flow, and the termination of the steady propagation of the FC process will occur. The point of the termination on the plot $\eta_f(\tilde{v}_0)$ corresponds to the left boundary of the 'peninsula' formed by the lines of two solutions, and at the adopted parameters of the system it is close to the line $\eta_f = \eta_{f,\min}$.

It should also be noted that at high values of κ and low filtration rates the non-uniqueness of the solutions for $\eta_f(\tilde{v}_0)$ has been found. Probably, the solution with the smaller value of η_f is an unstable one, and the fading of the process in the case of κ = const. and decreasing filtration rate is associated with the attainment of the point of joining of two solutions.

Opposing FC wave (Fig. 7). When $\kappa > 0$, a new 'low rate' branch of the solution appears as a result of which the function $\eta_f(\tilde{v}_0)$ is characterized by the presence of the minimum. The region of the existence of an opposing FC wave at κ = const., ξ_0 = const. is limited by two values of the filtration rate at which the complete conversion extent is achieved ($\eta_f = 1$). An increase in the heat losses narrows the range of filtration rates within which an opposing wave can exist (with $\eta_f < 1$), and at some value of $\kappa = \kappa_{cr}$ this regime is not realized at any filtration rate: $\kappa_{cr} \approx \eta_{cr_1}$. As to the conditions of steady combustion termination, then, in the considered FC regime at the given filtration rate, these are already characterized by the maximum possible conversion extent ($\eta_f = 1$) taking place at some maximum coefficient of heat losses $\kappa \leqslant \kappa_{cr}$.

In an opposing wave, with an increasing coefficient of heat loss, the heat balance is mainly brought about at the expense of a decrease in the speed of wave propagation and, to a much lesser degree, at the expense of a small decrease in the temperature of combustion. This feature is due to the fact that in such FC regimes the zone of reaction cools off giving heat to the initially cold substance 'incoming' with the velocity $\tilde{u}$. A decrease in the combustion wave speed is naturally accompanied by an increase in the conversion extent which attains its maximum at comparatively small values of κ.

It is of interest to consider the problem of mutual correspondence between the accompanying and opposing waves at $\xi_0 = \text{const.}$ (Fig. 8). It turns out that on the axis of filtration rates at $\kappa = \text{const.} < \kappa_{cr}$ there are: the region of existence of only accompanying inverse waves with $\eta_f < 1$, two regions of the same waves with $\eta_f = 1$ and a region having the solutions for an accompanying wave with $\eta_f = 1$ and for an opposing wave with $\eta_f < 1$ (Fig. 8a). Numerical simulation of the corresponding unsteady-state problem [23] has shown that in the case of reaction 'ignition' in the central portion of the specimen, an opposing wave develops in the last region of filtration rates. Finally, the presence of the factors characteristic of real processes (reaction retardation by the product, its zero order with respect to an oxidant, drop in the thermal conductivity of the system with an increase in the conversion extent) shifts to the left of the region of filtration rates, where the accompanying inverse FC waves with $\eta_f = 1$ may exist, and leads to an increasing role of heat losses and to a sharp decrease in the critical value κ_{cr_1} (Fig. 8b). For the regime with an opposing FC wave, the region of the solution non-uniqueness appears, with the branch that has the lower value of η_f turning to be stable; an increase in the level of heat losses leads to the termination of this wave not at the moment of complete conversion in it, but at somewhat lower values of η_f [23].

The results given have provided an extra proof to the conclusion drawn in Section 3 about a more complex character of the effect of heat losses on the propagation of FC waves with forced gas filtration as compared with the combustion of classical gas systems [25]. The reason for this lies in the presence of additional factors of heat transfer between the combustion wave and

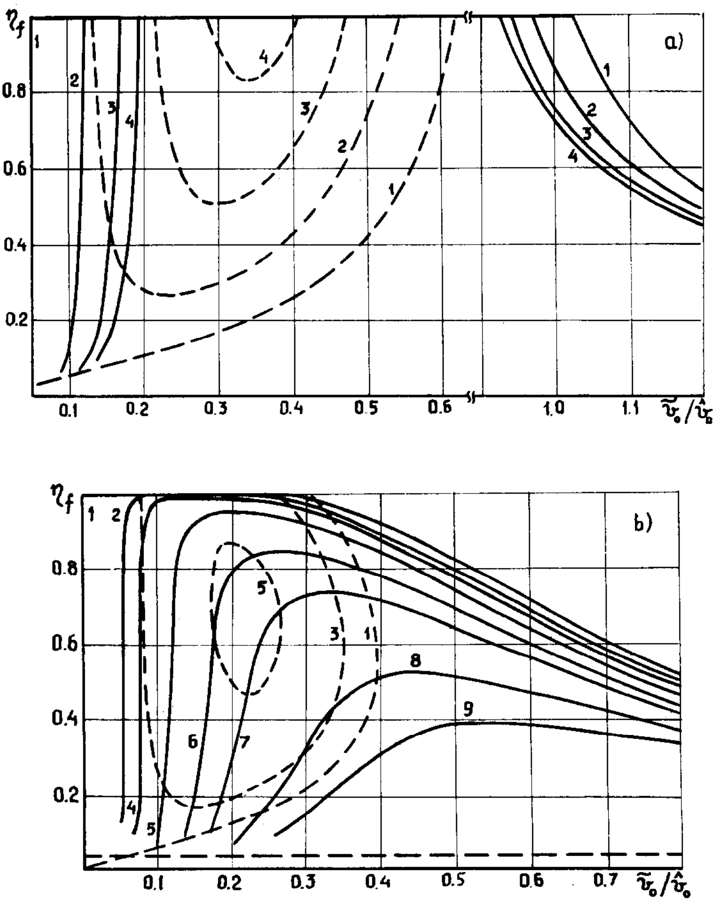

FIG. 8. The final conversion extent of the treated material vs gas mixture filtration rate at $\xi_0 = 0.031$ in the regimes of accompanying inverse (—— $\xi_0 = 1$) and opposing (--- $\xi_0 = 0.906$) FC waves. (a) $n = l = m = 0$; κ is equal to: 1, 0; 2, 0.001; 3, 0.00015; 4, 0.00018. (b) $n = l = 1/2$, $m = 0.9$, κ is equal to: 1, 0; 2, 0.00004; 3, 0.00005; 4, 0.00006; 5, 0.0001; 6, 0.00015; 7, 0.0002; 8, 0.0003; 9, 0.0004.

filtering gas and in the feasibility of regimes with incomplete conversion of the initial substance. However, some regularities noted in ref. [25] are also present in the case considered: (1) a limiting drop of the temperature in an accompanying inverse wave on an increase in heat losses, as compared with the temperature in a wave without heat losses and the same 'inlet' parameters, is equal to about 1.1 of the characteristic temperature interval; and (2) for an opposing wave the temperature drops by the same amount on the attainment of κ_{cr} relative to the temperature of an adiabatic wave with $\eta_f = 1$ and the same oxidant concentration ξ_0.

4. *Optimization of a low-temperature technological combustion process*

When using FC as the basis for this or other technological process, there naturally arises a question regarding its efficiency, which is ultimately determined by the speed of reaction wave propagation throughout a condensed phase. In this respect, there are two possibilities for the efficiency optimization. The first consists in the use of multi-wave regimes [28] in which the required conversion extent is achieved by successive passage of several reaction waves, and the second in the choice of the most efficient single-wave regime [29].

As a rule, the prescribed technological parameters of the process are the final conversion extent η_f and the temperature level of reaction T_c. In a single-wave regime these conditions (at a fixed inlet gas temperature T_0) uniquely determine both the choice of the inlet parameters of a gas flow v_0 and ξ_0, and the outlet characteristic which is the speed of reaction wave propagation u.

The possibility for optimization appears in the multiple passage of reaction waves with one and the same temperature T_c but with successively varying, after each passage of the wave, material conversion extent: $0, \eta_{f_1}, \eta_{f_2}, \ldots, \eta_{f_N} \equiv \eta_f$ (where N is the number of waves). This scheme of a technological process allows variation both of the number of waves and of the interval of variation of the conversion extent on passage of each of the waves to obtain the process of maximum efficiency.

Let u_{N_j} be the speed of the jth wave propagation in the N-wave regime. This speed is determined, as has been already noted, by the temperatures T_c, T_0 and by the range of conversion extent $[\eta_{f_{(j-1)}}, \eta_{f_j}]$. Then, the effective rate of the process with N waves is expressed as

$$u_N = 1\Big/\sum_{j=1}^{N} u_{N_j}^{-1} \tag{38}$$

and it may be used to obtain the maximum value at a certain optimum set of $N-1$ free parameters: $\eta_{f_1}, \eta_{f_2}, \ldots, \eta_{f_{(N-1)}}$.

The condition for the constant combustion temperature for the entire sequence of waves is quite natural for technological processes in which the reaction temperature is limited by reference to the yield of the target product. It allows one to rather simply obtain the expression for the efficiency of multi-wave FC regimes [28]. In the Zel'dovich–Frank-Kamenetsky's approximation (narrow reaction zone) [30] this expression has the form:

$$\frac{u_N}{u_1} = \frac{\left\{\int_0^{\eta_f} [(\tilde{\eta})^{1-n}(\tilde{\eta}+\bar{\eta}_f(1/\xi_0-1))^n/f(\eta)\lambda(\eta)]\,\mathrm{d}\tilde{\eta}\right\}^{1/2}}{\sum_{j=1}^{N}\left\{\int_0^{\eta_{f_j}-\eta_{f_{(j-1)}}} [(\tilde{\eta})^{1-n}(\tilde{\eta}+\bar{\eta}_f(1/\xi_0-1))^n/f(\eta)\lambda(\eta)]\,\mathrm{d}\tilde{\eta}\right\}^{1/2}} \tag{39}$$

where ξ_0 is defined by expression (19), $\eta = \tilde{\eta}+\eta_{f_{(j-1)}}$ for the case of an accompanying wave; $\eta = \eta_{f_j}-\tilde{\eta}$ for the case of an opposing wave; $\bar{\eta}_f = \eta_{f_j}-\eta_{f_{(j-1)}}$.

It follows from expression (39) that the functions $f(\eta)$ and $\lambda(\eta)$ exert the same qualitative influence on the ratio u_N/u_1. In the case of reactions without retardation by the product $[f(\eta) = 1]$ and constant effective thermal conductivity of the system $[\lambda(\eta) = \lambda_0]$, the use of multi-wave regimes gives no positive effects, i.e. $u_N \equiv u_1$. In all the other cases the efficiency of the multi-wave process can have an extremum with a certain set of $\{\eta_{f_j}\}$.

As an illustration, Fig. 9 presents the results of calculations by formula (39) for two-wave FC regimes with the model functions $f(\eta) = (1-\eta)^l$ and $\lambda(\eta) = \lambda_0(1-m\eta)$ and zero-order reaction. Here and hereafter, in all the calculations, the final conversion extent is taken to be equal to $\eta_f = 0.95$ (thus allowing the use of the narrow reaction zone approximation).

It is seen that the effectiveness of the two-wave regime as regards the technological process enhancement depends substantially on the form of the function $\lambda(\eta)$. In the case when the thermal conductivity of the system decreases or remains constant with an increase of the conversion extent, the intensification can be achieved in the mode of an accompanying wave (solid curves 1–4) and, conversely, in the case of thermal conductivity increase with η, the positive effect can be achieved (depending on the degree of reaction retardation) in the opposing wave state (dashed curve 5).

The physical reason for an increase in the effective rate of the process in two-wave or multi-wave modes is that, as follows from relation (39), at oxidant-based reaction orders smaller than 1 the greatest effect on the speed of wave propagation is exerted by the values of the thermal conductivity and retardation function that correspond to the beginning of the reaction zone which is located at a boundary with the final product where the temperature profiles of the reaction and heating zones join.

The analysis of the results of numerical minimization of the function of several variables [denominator in relation (39)] shows that in the case of a fractional

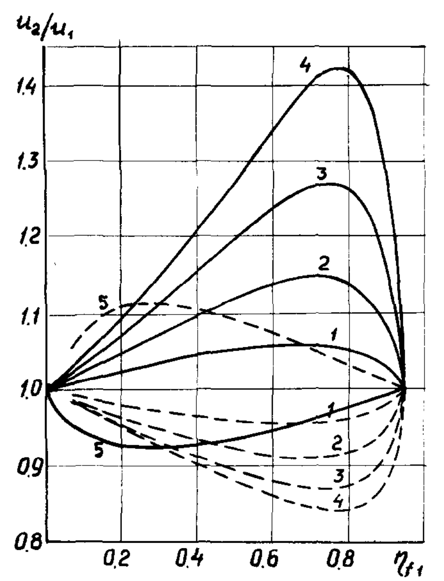

FIG. 9. The efficiency of two-wave FC regime as a function of the treated material conversion extent during the passage of the first wave. 1, $m = 0$, $l = 1/2$; 2, $m = 0$, $l = 1$; 3, $m = 1$, $l = 1/2$; 4, $m = 1$, $l = 1$; 5, $m = -10$, $l = 0$. —— accompanying waves; --- opposing waves.

reaction order the multi-wave regimes may turn out to be ineffective; this takes place, for example, in the absence of reaction retardation and at constant thermal conductivity. An increase of the retardation degree and the appearance of the declining function $\lambda(\eta)$ lead to an increase in the efficiency of multi-wave regimes.

With an increase in the number of waves the intensity of the process increases, but the rate of this increase rapidly slows down. Therefore, the use of accompanying wave regimes with more than three or four waves seems to be of no practical value.

A comparison of the opposing and accompanying FC regimes [29] has revealed the non-unique character of the assessment of their comparative efficiency ($u_{\text{opp}}/u_{\text{ac}}$). Depending on the value of η_f and on this or other combinations of system parameters that determine the order of reaction, degree of its retardation by the product, rising or declining character of the function $\lambda(\eta)$, the optimal (with respect to the process efficiency) turns to be either an opposing or an accompanying wave regime.

One should note an extremely strong effect of the reaction order on the dependence of $u_{\text{opp}}/u_{\text{ac}}$ on η_f. This dependence is monotonously increasing for the cases of a constant and decreasing thermal conductivity of the system at the zero reaction order; at $n = 1/2$ it has a minimum, and at $n \geqslant 1$ becomes decreasing.

In the case of λ being independent of η or in the case of a decreasing dependence of λ on η and at the reaction orders from $n = 0$ to $n = 1$, the accompanying FC regime is more efficient than an opposing one for the entire range of the possible values of the final conversion extent. The efficiency of its application increases in transition from systems with λ = const. to the systems with a declining function $\lambda(\eta)$ and stronger reaction retardation. In the case of the higher dependence of the reaction rate on oxidant concentration ($n > 1$) for the above-mentioned types of the function $\lambda(\eta)$ the opposing FC regime turns to be more effective over the entire range of η_f values only in the case of the absence or slight retardation of reaction. In the case of stronger retardation, this regime starts to give place to the accompanying FC regime with the growth of η_f; the earlier this transition occurs the stronger is the retardation.

For systems with a rising function $\lambda(\eta)$ for a certain (mean) range of the values of η_f, the accompanying FC regime turns out to be more effective and the more so, the weaker is reaction retardation by the product.

In conclusion it should be noted that additional 'exact' (numerical) solutions of the system of equations (1)–(4) have shown the validity of the use of the narrow reaction zone approximation in calculations of u_N/u_1 and $u_{\text{opp}}/u_{\text{ac}}$.

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COMBUSTION A BASSE TEMPERATURE DE SYSTEMES POREUX AVEC FILTRATION FORCEE D'UN GAZ REACTANT

Résumé—En utilisant la simulation numérique d'une approximation monodimensionnelle adiabatique, on étudie la combustion permanente et la stabilité dans les modes d'ondes à réaction. En considérant les propriétés réelles d'un système, les formules sont établies entre les caractéristiques de l'onde de combustion (température, vitesse de propagation de l'onde et extension de la phase condensée dans l'onde) et les paramètres gouvernant les mécanismes: vitesse de filtration de mélange de gaz, température et concentration d'un composant actif (oxydant). L'effet des pertes de chaleur sur la propagation de ces ondes est considéré en détail. L'optimisation des régimes de combustion (oxydant). L'effet des pertes de chaleur sur la propagation de ces ondes est considéré en détail. L'optimisation des régimes de combustion (incluant ceux avec des passages multiples d'ondes) est traitée en considérant la performance d'une unité technologique.

TECHNOLOGISCHE NIEDERTEMPERATURVERBRENNUNG IN PORÖSEN SYSTEMEN MIT ERZWUNGENER FILTRATION EINES GASREAKTANTEN

Zusammenfassung—Die Gesetzmäßigkeiten der stationären Filtrationsverbrennung im Fall von begleitenden inversen und entgegengesetzten Reaktionswellen werden mit Hilfe einer numerischen Simulation, einer eindimensionalen adiabatischen Approximation, untersucht. Unter Betrachtung einiger realer Eigenschaften des Systems wurden die Zusammenhänge zwischen den Charakteristiken der Verbrennungswellen (Temperatur und Geschwindigkeit der Wellenfortschreitung und Ausmaß der Konvertierung der kondensierten Phase in der Welle) und den bestimmenden Parametern des Prozesses festgestellt: die Filtrationsgeschwindigkeit des Gasgemisches, die Temperatur und die Konzentration der aktiven Komponenten (Oxidanten). Der Einfluß der Wärmeverluste auf die Fortschreitung dieser Wellen wird im Detail erörtert. Die Optimierung der Filtrationsverbrennungsregime (einschließlich solcher mit mehrfachem Reaktionswellendurchgang) unter Berücksichtigung des Verhaltens eines auf Verbrennung basierenden technologischen Prozesses wird durchgeführt.

НИЗКОТЕМПЕРАТУРНОЕ ТЕХНОЛОГИЧЕСКОЕ ГОРЕНИЕ ПОРИСТЫХ СИСТЕМ С ВЫНУЖДЕННОЙ ФИЛЬТРАЦИЕЙ ГАЗА-РЕАГЕНТА

Аннотация—Численным моделированием в одномерном адиабатическом приближении изучены закономерности стационарного фильтрационного горения в режимах спутной инверсной и встречной волн реакции. С учетом ряда реальных свойств системы установлены зависимости характеристик волны горения (температуры, скорости распространения волны и глубины превращения в ней конденсированной фазы) от управляющих параметров процесса: скорости фильтрации газовой смеси, ее температуры и концентрации в ней активного компонента (окислителя). Детально рассмотрено влияние теплопотерь на распространение указанных волн. Проведена оптимизация режимов фильтрационного горения (включая режимы с многократным прохождением волны реакции) по производительности основанного на нем технологического процесса.